1.7 Videos Guide

1.7a

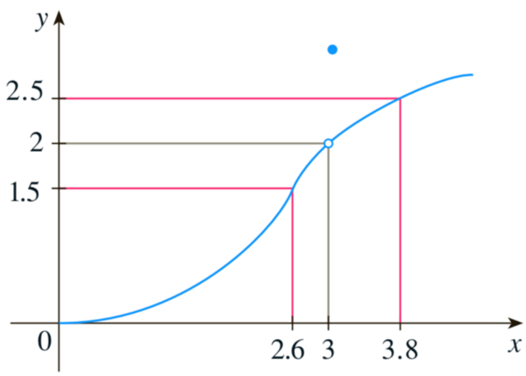
Definition: (limit)

- Let f be a function defined on an open interval containing a (except possibly at a). Then $\lim_{x \to a} f(x) = L$ if for every $\varepsilon > 0$ there is a number $\delta > 0$ such that if $0 < |x - a| < \delta$ then $|f(x) - L| < \varepsilon$.

1.7b

Exercise:

- Use the graph of f to find a number δ such that if $0 < |x - 3| < \delta$ then $|f(x) - 2| < 0.5$.



Exercises:

1.7c

- Find L such that $\lim_{x \to 2}(5x - 7) = L$. Then find δ-values that correspond to $\varepsilon = 0.1$, $\varepsilon = 0.05$, and $\varepsilon = 0.01$.

1.7d

- Prove the statement using the ε, δ definition of a limit.
  - $\lim_{x \to 10}\left(3 - \frac{4}{5}x\right) = -5$

1.7e

Exercise:

  - $\lim_{x \to 2}(x^2 + 2x - 7) = 1$

Proofs:

1.7f

- Sum Limit Law: If $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist, then $\lim_{x \to a}[f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$.

1.7g

- The Squeeze Theorem: If $f(x) \leq g(x) \leq h(x)$ on some interval containing a (except possibly at a) and if $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$, then $\lim_{x \to a} g(x) = L$.